IN-34
181585
p. 21

NASA Contractor Report 191487

ICASE Report No. 93-33

ICASE

20 Years of Excellence

THE EFFECT OF CROSSFLOW ON TAYLOR VORTICES: A MODEL PROBLEM

S. R. Otto

Andrew P. Bassom

(NASA-CR-191487) THE EFFECT OF CROSSFLOW ON TAYLOR VORTICES: A MODEL PROBLEM Final Report (ICASE) 21 p

N94-13128

Unclas

G3/34 0181585

NASA Contract No. NAS1-19480

June 1993

Institute for Computer Applications in Science and Engineering

NASA Langley Research Center

Hampton, Virginia 23681-0001

Operated by the Universities Space Research Association

National Aeronautics and Space Administration

Langley Research Center

Hampton, Virginia 23681-0001

ICASE Fluid Mechanics

Due to increasing research being conducted at ICASE in the field of fluid mechanics, future ICASE reports in this area of research will be printed with a green cover. Applied and numerical mathematics reports will have the familiar blue cover, while computer science reports will have yellow covers. In all other aspects the reports will remain the same; in particular, they will continue to be submitted to the appropriate journals or conferences for formal publication.

The effect of crossflow on Taylor vortices: a model problem

S. R. Otto[1†] and Andrew P. Bassom[2],

[1] *ICASE, Mailstop 132c, NASA Langley Research Center, Hampton, VA23681–0001, USA.*

[2] *Department of Mathematics, North Park Road, University of Exeter, Exeter, Devon. EX4 4QE. U.K.*

Abstract

A number of practically relevant problems involving the impulsive motion or the rapid rotation of bodies immersed in fluid are susceptible to vortex–like instability modes. Depending upon the configuration of any particular problem the stability properties of any high–wavenumber vortices can take on one of two distinct forms. One of these is akin to the structure of Görtler vortices in boundary layer flows whilst the other is similar to the situation for classical Taylor vortices.

Both the Görtler and Taylor problems have been extensively studied when crossflow effects are excluded from the underlying base flows. Recently, studies have been made concerning the influence of crossflow on Görtler modes and here we use a linearised stability analysis to examine crossflow properties for the Taylor mode. This work allows us to identify the most unstable vortex as the crossflow component increases and we show how, like the Görtler case, only a very small crossflow component is required in order to completely stabilise the flow. Our investigation forms the basis for an extension to the nonlinear problem and is of potential applicability to a range of pertinent flows.

† Research was supported by the National Aeronautics and Space Administration under NASA contract No. NAS1–19480 while the author was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23681

1. Introduction

The problem of predicting vortex breakdown is of great importance within the field of transition research. In many practical situations it is desirable to influence breakdown characteristics: for example in aerospace design the requirement is that transition be delayed as long as possible in order to reduce drag whereas in jet intakes turbulence is promoted in order to improve fuel mixing. In numerous applications where flow curvature is present the stability problem is closely related to structures describing Taylor and/or Görtler modes at large Taylor/Görtler numbers. Undoubtedly in many cases the experimental flow is fully three–dimensional in character and until recently little theoretical work had been conducted on such problems.

The linear stability of *short* wavelength Taylor–Görtler vortices was examined by Hall **[1]** for both fully developed and boundary layer flows. The model problem considered was the flow contained between a pair of concentric cylinders whose separation is much smaller than the radii of the cylinders. A WKB type analysis shows that in the Dean problem (in which both cylinders are at rest and the fluid is driven by a constant azimuthal pressure gradient) short wavelength vortices are constrained to lie in a thin layer located at a known position between the cylinders. In contrast, for the Taylor problem (in which the outer cylinder is at rest and the inner cylinder rotates uniformly) the vortex structure is trapped at the inner cylinder wall. These two idealised cases capture the essence of a large number of important practical flows. For example, the Dean structure forms the basis for describing the evolution of Görtler vortices in spatially varying boundary layers (although extra non–parallel effects do need to be considered) whereas the wall–bounded vortices are relevant to a number of impulsively started spin–up problems, **[2]**. The linear stability theories for the Taylor and Dean cases have been extended to weakly and fully nonlinear accounts, **[3-6]**.

The first work concerned with the effect of adding a crossflow component to the basic flow was conducted by Hall, **[7]**. He showed that only a very small crossflow velocity is needed to radically alter the linear stability properties of short wavelength Görtler vortices. In many practical situations interest cannot be legitimately restricted to high wavenumber modes alone and, on a linear stability basis, it is frequently the fastest growing (or most unstable) vortex which is likely to be important. That this mode is of interest is clear for often there will be a whole spectrum of vortex wavenumbers that may be excited in any one particular problem and the fastest growing of these is the one that is dominant. The most unstable Görtler mode within a two–dimensional boundary layer was identified by Denier *et. al.* **[8]** and the linear and nonlinear properties of this mode as the crossflow component is increased have been developed in **[9-11]**. To the best of our knowledge the most unstable vortex in the Taylor problem has yet to be noted.

The purpose of this concise article is to outline the effect of crossflow on vortices in

the Taylor situation for cylinders with a small separation. In particular we shall describe the influence of crossflow on high wavenumber modes, specify the structure of the most unstable vortex and track the linear stability properties of this mode as the crossflow grows. It may be argued that this problem is very specific and therefore is of little, or no, practical use. However we should emphasise that the analysis here encompasses the ideas behind a large number of important cases including problems involving rapidly rotating bodies and impulsive motion. Thus the merit of using the classical small gap Taylor problem as a paradigm is that the analysis is very straightforward, enables the important features of the flow development to be seen and our workings may be easily generalised to account for more complicated situations.

The remainder of the paper is organised as follows. In §2 we describe the relevant stability equations and in §3 show how increasing crossflow modifies the large wavenumber structures. The most unstable mode is obtained in §4 and we then outline the influence of crossflow on this vortex. We close with a brief discussion.

2. The governing stability equations

The linearised disturbance equations pertaining to Taylor vortices have been derived many times and so here we restrict ourselves to the briefest of descriptions of these equations. Consider the flow of incompressible fluid of density ρ and kinematic viscosity ν in the gap between a pair of concentric cylinders of radii R_1 and R_2 ($> R_1$). We assume that the curvature of the gap is small; that is $\delta \equiv (R_2 - R_1)/R_1 \ll 1$. The inner cylinder rotates with angular velocity U_0/R_1 whilst the outer cylinder is at rest. It is convenient to define variables x, y, z, t by

$$x = \frac{Re\theta'}{\delta}, \qquad y = \frac{r' - R_1}{R_2 - R_1}, \qquad z = \frac{z'}{R_2 - R_1}, \qquad t = \frac{U_0 t'}{Re(R_2 - R_1)}, \tag{1}$$

where (r', θ', z') are the usual cylindrical polar co-ordinates, t' is the time variable and $Re \equiv U_0(R_2 - R_1)/\nu \gg 1$ is the Reynolds number. Writing the velocity field (scaled on the characteristic fluid speed U_0) with respect to (x, y, z) as (u, v, w) and scaling the pressure p on ρU_0^2 gives that in the basic state

$$(u, v) \equiv (\bar{u}, \bar{v}) = (1 - y, 0) + O(\delta). \tag{2a}$$

In order to introduce a crossflow component into the flow we suppose that a constant axial pressure gradient acts and the size of this gradient is chosen so that the crossflow affects the subsequent stability equations. It is a routine calculation to show that this occurs when the axial velocity component $\bar{w} = O(Re^{-1})$ and then the remainder of the basic state solution is

$$(w, p) \equiv (\bar{w}, \bar{p}) = \lambda \left(\frac{1}{Re} \left\{ y(1 - y) + O(\delta) \right\}, -\frac{1}{Re^2} \left\{ 2z + O(\delta) \right\} \right); \tag{2b}$$

where we have introduced λ as a measure of the strength of the scaled axial flow. The situation discussed in this article has been studied by many prolific authors, the reader is referred to **[12]** for further reading. The earliest theoretical and experimental studies were apparently made by **[13]** and **[14]** respectively.

The Taylor number $T \equiv 2Re^2\delta$ is taken to be an O(1) parameter and we perturb the basic state by writing

$$(u, v, w, p) = (\bar{u}, 0, \bar{w}, \bar{p}) + \Delta \left\{ U(x, y, z, t), \frac{1}{Re} V(x, y, z, t), \frac{1}{Re} W(x, y, z, t), \frac{1}{Re^2} P(x, y, z, t) \right\}, \tag{3}$$

where Δ is a vanishingly small parameter. If we substitute (3) into the linearised forms of the continuity and Navier–Stokes equations and neglect terms of relative size $O(\delta)$, we are left with the four disturbance equations

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0, \tag{4a}$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t} \right) U = (1-y)\frac{\partial U}{\partial x} - V + \lambda y(1-y)\frac{\partial U}{\partial z}, \tag{4b}$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t} \right) V - \frac{\partial P}{\partial y} = (1-y)\frac{\partial V}{\partial x} + \lambda y(1-y)\frac{\partial V}{\partial z} - T(1-y)U, \tag{4c}$$

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t} \right) W - \frac{\partial P}{\partial z} = (1-y)\frac{\partial W}{\partial x} + \lambda(1-2y)V + \lambda y(1-y)\frac{\partial W}{\partial z}, \tag{4d}$$

which need to be solved subject to the no–slip conditions $U = V = W = 0$ on $y = 0, 1$. In the absence of crossflow ($\lambda = 0$) and with no azimuthal dependence ($\partial/\partial x \equiv 0$) system (4) reduces to an ordinary differential eigenvalue problem for the neutral Taylor number as a function of vortex wavenumber a. It is well known that for $a \ll 1$ then $T = O(a^{-2})$ whilst, for large a, $T = O(a^4)$. We now analyse the effect of crossflow on this right–hand branch of the neutral stability curve; i.e. on neutral large–wavenumber vortex structures.

3. The effect of crossflow on large wavenumber modes

It was shown in **[1]** that large wavenumber vortices in a flow with zero axial component are trapped in a thin zone of depth $O(a^{-\frac{2}{3}})$ next to the inner cylinder wall $y = 0$. Analysis of equations (4) reveals that when the scaled crossflow size $\lambda = O(a)$ this wall structure needs to be adjusted and consequently we shall examine this case now. In the neighbourhood of the neutral stability curve it is known that the vortices have temporal growth rate $O(a^2)$ and a waviness in the azimuthal direction on an $O(a^{-\frac{4}{3}})$ lengthscale. Using the scalings proposed in **[1]** we write the Taylor number

$$T = T_0 a^4 + T_1 a^{\frac{10}{3}} + \dots, \tag{5a}$$

seek modes proportional to

$$E_1 \equiv \exp\left[iaz + a^{\frac{4}{3}}(\sigma_0 + a^{-\frac{2}{3}}\sigma_1 + \ldots)x + a^2 \int^t (\Omega_0(t) + a^{-\frac{2}{3}}\Omega_1(t) + \ldots)dt\right], \qquad (5b)$$

and allow the crossflow size $\lambda = a\hat{\lambda}$. If we further define the O(1) co-ordinate ξ within the wall layer by

$$y = a^{-\frac{2}{3}}\xi, \qquad (6a)$$

then the disturbance quantities (U, V, W, P) develop according to

$$(U, V, W, P) = \left(U_0 + a^{-\frac{2}{3}}U_1 + \ldots, a^2 V_0 + a^{\frac{4}{3}}V_1 + \ldots, a^{\frac{5}{3}}W_0 + aW_1 + \ldots, a^{\frac{8}{3}}P_0 + a^2 P_1 + \ldots\right) E_1. \qquad (6b)$$

Substitution of (5) & (6) into $(4b, c)$ yields at leading orders

$$-(1+\Omega_0)U_0 = -V_0, \qquad -(1+\Omega_0)V_0 = -T_0 U_0,$$

which are consistent only if

$$T_0 = (1+\Omega_0)^2. \qquad (7)$$

In order to determine the structure of the leading order disturbance functions it is necessary to proceed to higher orders in (4). These give the governing equation

$$(\Omega_0 + 3)\frac{d^2 V_0}{d\xi^2} - V_0\left[\xi\left(1 + \Omega_0 + 2i\hat{\lambda}\right) - \left(\frac{T_1}{T_0}(1+\Omega_0) - 2(\Omega_1 + \sigma_0)\right)\right] = 0,$$

which is a scaled version of the Airy equation and has solution

$$V_0 \propto Ai\left[\left(\frac{1+\Omega_0+2i\hat{\lambda}}{3+\Omega_0}\right)^{\frac{1}{3}}\xi - C\right], \qquad (8a)$$

where

$$C = \frac{(\Omega_0+3)^{-\frac{1}{3}}}{(1+\Omega_0+2i\hat{\lambda})^{\frac{2}{3}}}\left(\frac{T_1}{T_0}(1+\Omega_0) - 2(\Omega_1+\sigma_0)\right). \qquad (8b)$$

In order to satisfy the inviscid constraint that $V_0 \to 0$ as $\xi \to 0$ it is necessary to choose C so that $Ai(-C) = 0$ whence

$$C \approx 2.3381 \qquad (8c)$$

and then $(8b)$ may be rearranged to give the Taylor number correction T_1. Although $V_0 \to 0$ as $\xi \to 0$ the continuity equation $(4a)$ gives $W_0 = i(dV_0/d\xi)$ and so a passive viscous layer is required to ensure that the axial disturbance velocity component vanishes on the inner cylinder. The details of this layer are very elementary so will not be pursued here.

Definition (5b) shows that for neutrally stable large wavenumber modes the frequency terms Ω_0, Ω_1 are necessarily purely imaginary. Thence result (7) gives $T_0 = 1$ and (8b) shows that

$$T_1 = 3^{\frac{1}{3}} C(1 + 4\hat{\lambda}^2)^{\frac{1}{3}} \cos\left(\tfrac{1}{3}\tan^{-1}(2\hat{\lambda})\right). \tag{9}$$

Consequently for large scaled crossflow $\hat{\lambda}$, $T_1 = O(\hat{\lambda}^{\frac{2}{3}})$ and expansion (5a) implies that when $\hat{\lambda} \sim a$ the neutral Taylor number is altered by an $O(a^4)$ amount from its value in the zero crossflow state. Furthermore, the extent of the vortex activity is compressed into a thinner, $O(a^{-1})$, depth region adjacent to $y = 0$ and so we write

$$\lambda = a^2 \tilde{\tilde{\lambda}}, \qquad y = a^{-1}\chi, \tag{10a,b}$$

$$E_2 \equiv \exp\left[iaz + a^2(\tilde{\tilde{\sigma}}_0 + \ldots)x + a^2 \int^t (\tilde{\tilde{\Omega}}_0(t) + \ldots)dt\right] \tag{10c}$$

and write the vortex in the form

$$(U, V, W, P) = \left(\tilde{\tilde{U}}_0 + \ldots, a^2\tilde{\tilde{V}}_0 + \ldots, a^2\tilde{\tilde{W}}_0 + \ldots, a^3\tilde{\tilde{P}}_0 + \ldots\right) E_2. \tag{10d}$$

Substitution into (4) leads to the disturbance equations

$$\begin{gathered}\left(\frac{d^2}{d\chi^2} - 1 - \tilde{\tilde{\Omega}}_0 - \tilde{\tilde{\sigma}}_0 - i\tilde{\tilde{\lambda}}\chi\right)\left(\frac{d^2}{d\chi^2} - 1\right)\tilde{\tilde{V}}_0 = T_0\tilde{\tilde{U}}_0, \\ \left(\frac{d^2}{d\chi^2} - 1 - \tilde{\tilde{\Omega}}_0 - \tilde{\tilde{\sigma}}_0 - i\tilde{\tilde{\lambda}}\chi\right)\tilde{\tilde{U}}_0 = -\tilde{\tilde{V}}_0,\end{gathered} \tag{11}$$

which need to be solved subject to $\tilde{\tilde{U}}_0 = \tilde{\tilde{V}}_0 = d\tilde{\tilde{V}}_0/d\chi = 0$ at $\chi = 0$ and as $\chi \to \infty$.

At this point it is worthwhile to recall the results of [7] which was concerned with describing the effect of crossflow on the neutral stability properties of high wavenumber Görtler vortices. In the absence of crossflow in that case the mode is confined to a thin layer away from the bounding surface. Three distinctive sizes of crossflow were identified. In the first the decay of the vortices away from the thin layer is primarily due to the fact that the local Görtler number is maximised at the centre of the layer; at a higher crossflow the decay is driven by convective effects and in the third regime the layer is driven to the wall and the vortex is governed by a pair of viscous equations similar in style to (11) but containing quadratic terms as well. Thus in the Görtler (or equivalently the Dean) problem the evolution of the vortex with increasing crossflow passes through three phases whereas for the Taylor problem studied here we have only two stages.

Equations (11) were solved using a finite difference technique and, in order to capture neutral modes, the quantities $\tilde{\tilde{\Omega}}_0$ and $\tilde{\tilde{\sigma}}_0$ were taken as purely imaginary. The results expressing T_0 and $(\tilde{\tilde{\Omega}}_0 + \tilde{\tilde{\sigma}}_0)$ as functions of the scaled crossflow $\tilde{\tilde{\lambda}}$ are shown in figure 1. Of interest is the solution of (11) in the limits of large or small crossflows. As $\tilde{\tilde{\lambda}} \to 0$ then

we find that $T_0 \to 1$ and $\mathrm{Im}\,(\tilde{\tilde{\Omega}}_0 + \tilde{\tilde{\sigma}}_0) \to 0$ in agreement with the large $\hat{\lambda}$ results for the case presented above. When $\tilde{\tilde{\lambda}} \to \infty$ the solution divides into a double boundary layer structure. We postulate that

$$T_0 \sim d\tilde{\tilde{\lambda}}^2 + \ldots, \qquad \tilde{\tilde{\Omega}}_0 + \tilde{\tilde{\sigma}}_0 \sim ic\tilde{\tilde{\lambda}}^{\frac{2}{3}} + \ldots, \tag{12}$$

for some real constants c and d. In the majority of the region where $\chi = O(1)$ then

$$\tilde{\tilde{U}}_0 = \tilde{\tilde{\lambda}}^{-1} U_0^{\dagger} + \ldots, \qquad \tilde{\tilde{V}}_0 = V_0^{\dagger} + \ldots, \tag{13a}$$

where

$$-i\chi\left(\frac{d^2}{d\chi^2} - 1\right) V_0^{\dagger} = dU_0^{\dagger}, \qquad i\chi U_0^{\dagger} = V_0^{\dagger} \tag{13b}$$

and the solution of this system which decays exponentially as $\chi \to \infty$ is

$$(U_0^{\dagger}, V_0^{\dagger}) = \left(\chi^{-\frac{1}{2}} K_\nu(\chi), i\chi^{\frac{1}{2}} K_\nu(\chi)\right), \tag{14}$$

where K_ν is the modified Bessel function of order $\nu = \sqrt{d + \frac{1}{4}}$ (see **[15]**). As $\chi \to 0$ these solutions develop singularities so that in a wall layer where $\chi = \tilde{\tilde{\lambda}}^{-\frac{1}{3}} \tilde{Y}$, $\tilde{Y} = O(1)$, we write

$$\tilde{\tilde{U}}_0 = \tilde{\tilde{\lambda}}^{\frac{\nu}{3} + \frac{5}{6}} U_0^{\ddagger}(\tilde{Y}) + \ldots, \qquad \tilde{\tilde{V}}_0 = \tilde{\tilde{\lambda}}^{\frac{\nu}{3} - \frac{1}{6}} V_0^{\ddagger}(\tilde{Y}) + \ldots. \tag{15a, b}$$

These functions satisfy the coupled equations

$$\left(\frac{d^2}{d\tilde{Y}^2} - ic - i\tilde{Y}\right) \frac{d^2 V_0^{\ddagger}}{d\tilde{Y}^2} = dU_0^{\ddagger}, \qquad \left(\frac{d^2}{d\tilde{Y}^2} - ic - i\tilde{Y}\right) U_0^{\ddagger} = -V_0^{\ddagger}, \tag{15c, d}$$

which need to be solved subject to the conditions that $U_0^{\ddagger} = V_0^{\ddagger} = dV_0^{\ddagger}/d\tilde{Y} = 0$ on $\tilde{Y} = 0$ together with suitable matching conditions to (14) as $\tilde{Y} \to \infty$. Numerical solution of this problem reveals that

$$c = -4.706, \qquad d = 0.721, \tag{16}$$

and the asymptotic forms (12) for T_0 and $\tilde{\tilde{\Omega}}_0 + \tilde{\tilde{\sigma}}_0$ are sketched on figure 1. As may be seen, agreement between the numerical solution of (11) and these asymptotes is eminently satisfactory and demonstrates that as the (scaled) crossflow moves through this $O(a^2)$ regime the changes induced in the neutral Taylor number are substantial. Therefore only a very small axial field of size $O(Re^{-1}a^2)$ is required in order to radically distort the neutral curve for large wavenumber modes and so suggests that any such wavenumber vortex would be completely stabilised by a tiny crossflow.

In many practical situations Taylor/ Görtler numbers are large and, whilst the effect of any crossflow component on the neutral curve is of interest, it is also important to describe

how the crossflow modifies the growth rate of the most unstable vortex. As was mentioned in the introduction, the most unstable mode is frequently the one that is dominant in practice and, as far as we are aware, the location of this mode has not been written down for the Taylor problem. Therefore in the coming section we first obtain the most unstable vortex in the absence of crossflow and then consider how the imposition of a gradually increasing crossflow component modifies the stability properties of this disturbance.

4. The most unstable mode

In order to deduce the most unstable linear vortex in the absence of crossflow it is easiest to proceed in the manner of Denier *et. al.* **[8]**. In this work the authors considered both the growth rate of inviscid (O(1) wavenumber) Görtler modes as the wavenumber increases and also the properties of modes in the vicinity of the right hand branch of the linear neutral curve. It was then possible to identify an intermediate wavenumber regime in which infinitesimal high Görtler number vortices are the fastest growing.

The inviscid problem relevant to Taylor vortices for $T \gg 1$ has been considered numerous times (see **[16]** for example) and for vortex wavenumber $a = O(1)$ the disturbance takes the form

$$(U, V, W, P) =$$
$$(U_0(y) + \ldots, T^{\frac{1}{2}} V_0(y) + \ldots, T^{\frac{1}{2}} W_0(y) + \ldots, TP_0(y) + \ldots) \exp\left\{iaz + T^{\frac{1}{2}} \int^t \Omega(t)dt\right\}.$$

Substitution of these forms into the governing equations (4) shows that V_0 satisfies

$$\Omega^2 \left(\frac{d^2 V_0}{dy^2} - a^2 V_0\right) = -a^2(1-y)V_0, \tag{17}$$

subject to the boundary conditions that $V_0 = 0$ at $y = 0, 1$. This gives an eigenproblem for the growth rate Ω as a function of a. In order to locate the most unstable vortex across the entire wavenumber spectrum it is useful to consider the behaviour of the solution to (17) as $a \to \infty$. Elementary analysis (see **[2]**) shows that the eigenfunction V_0 is confined to a thin zone of depth $O(a^{-\frac{2}{3}})$ attached to $y = 0$. Thence

$$\Omega = 1 - \tfrac{1}{2} C a^{-\frac{2}{3}} + \ldots, \tag{18}$$

where the constant C satisfies $Ai(-C) = 0$ and has been given in ($8c$). Thus as $a \to \infty$ the growth rate of the vortex asymptotes

$$T^{\frac{1}{2}} \left(1 - \tfrac{1}{2} C a^{-\frac{2}{3}} + \ldots\right). \tag{19}$$

Otto **[2]** obtained this result for his problem concerning the impulsive spin-up of a cylinder within an expanse of fluid and we conclude that for large a the growth rate becomes independent of wavenumber to leading order.

Results ($5a$) and (7) demonstrate that at large vortex wavenumbers the right hand branch of the neutral stability curve is given by $T = a^4 + \ldots$. Then for $T \gg 1$ if we examine a vortex of wavenumber $a = \tilde{a}T^{\frac{1}{4}} + \ldots$ with $\tilde{a} = O(1)$ it is straightforward to deduce that the vortex is confined to a region of depth $O(T^{-\frac{1}{8}})$ next to the inner cylinder wall. Furthermore, the sizes of the disturbance components (U, V, W, P) are in the ratios $(1, T^{\frac{1}{2}}, T^{\frac{5}{12}}, T^{\frac{2}{3}})$ and the leading order vortex growth rate term is $O(T^{\frac{1}{2}})$. If we let this growth rate $\Omega = T^{\frac{1}{2}}\hat{\Omega} + \ldots$ then the compatibility criterion arising from the leading order balances from momentum equations ($4b, c$) reduces to

$$\hat{\Omega} = 1 - \tilde{a}^2.$$

Thus, as would be expected, the vortices are unstable for $\tilde{a} < 1$ and stable for $\tilde{a} > 1$. In addition, the overall growth rate is

$$T^{\frac{1}{2}}\left(1 - \tilde{a}^2\right) + \ldots \tag{20}$$

and thus the expressions (19) and (20) become comparable when

$$\tilde{a}^2 = \left(\frac{a}{T^{\frac{1}{4}}}\right)^2 \sim a^{-\frac{2}{3}} \qquad \Longrightarrow \qquad a \sim T^{\frac{3}{16}}.$$

This suggests that the most unstable Taylor vortex has wavenumber $O(T^{\frac{3}{16}})$ and we shall confirm this presently. Meanwhile we note how the geometry of growth rate/wavenumber space for the Taylor problem contrasts with that for the Görtler (or Dean) case. We have seen from (19), (20) that for a large wavenumber range, specifically $1 \ll a \ll T^{\frac{1}{4}}$, the growth rate of the Taylor mode is constant at leading order and only varies in higher order terms. Conversely, in the Görtler case the *leading* order growth rate increases both as $a \to \infty$ in the inviscid problem and as $\tilde{a} \to 0$ in the right–hand branch case. Therefore in the intermediate regime where the growth rate is greatest the maximum is a much more pronounced feature. This difference in growth rate behaviours for the two problems does have important consequences for the subsequent analysis. For the Görtler case the calculations of Bassom & Hall **[9]** show that steadily increasing crossflow stabilises the vortices and only a small crossflow is required in order to ensure that the flow is stable to infinitesimal modes irrespective of their wavelength. In addition it was found that as the crossflow grows so the location of the most unstable vortex remains in the same wavenumber regime: behaviour which is not repeated for the Taylor case discussed below.

In order to prove that it is the $O(T^{\frac{3}{16}})$ wavenumber regime which contains the most unstable Taylor mode we write

$$a = \hat{a}_0 T^{\frac{3}{16}}, \tag{21a}$$

and seek disturbances of the form

$$(U, V, W, P) = \left(\hat{U}_0 + \ldots, T^{\frac{1}{2}}\hat{V}_0 + \ldots, T^{\frac{7}{16}}\hat{W}_0 + \ldots, T^{\frac{3}{4}}\hat{P}_0 + \ldots\right) E_3, \tag{21b}$$

where $$E_3 \equiv \exp\left[i\hat{a}_0 T^{\frac{3}{16}} z + T^{\frac{1}{2}} \int^t \left(\hat{\Omega}_0(t) + T^{-\frac{1}{8}}\hat{\Omega}_1(t) + \dots\right) dt\right], \qquad (21c)$$

and $\hat{U}_0, \hat{V}_0, \dots$ are functions of $\eta \equiv T^{\frac{1}{8}} y$. Therefore these vortices are trapped in an $O(T^{-\frac{1}{8}})$ thick zone adjacent to $y = 0$ and we shall impose a crossflow of scaled size

$$\lambda = T^{\frac{5}{16}} \hat{\lambda}. \qquad (22)$$

This scaling is arrived at by choosing sufficient crossflow so as to perturb the vortex structure relevant to the zero crossflow problem without grossly changing its underlying characteristics. We substitute (21), (22) into equations (4) and compare like powers of T. The consistency condition arising from momentum balances $(4b, c)$ leads to

$$\hat{\Omega}_0 = 1, \qquad (23)$$

as would be anticipated from result (20). At next order we find that $\hat{V}_0$ satisfies the scaled Airy equation

$$\frac{d^2 \hat{V}_0}{d\eta^2} - \hat{a}_0^2(1 + 2i\hat{\lambda}\hat{a}_0)\left[\eta + \frac{2(\hat{a}_0^2 + \hat{\Omega}_1)}{(1 + 2i\hat{\lambda}\hat{a}_0)}\right]\hat{V}_0 = 0.$$

Demanding that $\hat{V}_0 = 0$ at $\eta = 0$ and as $\eta \to \infty$ leads to an expression for $\hat{\Omega}_1$ and thence we deduce the overall vortex growth rate $Gr = T^{\frac{1}{2}} + \hat{\Omega}_1 T^{\frac{3}{8}} + o(T^{\frac{3}{8}})$ where

$$\hat{\Omega}_1 = -\frac{C(1 + 4\hat{a}_0^2\hat{\lambda}^2)^{\frac{1}{3}}}{2\hat{a}_0^{\frac{2}{3}}}\left\{\cos\left(\tfrac{2}{3}\tan^{-1}(2\hat{a}_0\hat{\lambda})\right) + i\,\sin\left(\tfrac{2}{3}\tan^{-1}(2\hat{a}_0\hat{\lambda})\right)\right\} - \hat{a}_0^2. \qquad (24)$$

In the absence of the crossflow component $Re(\hat{\Omega}_1) = -\frac{1}{2}C\hat{a}_0^{-\frac{2}{3}} - \hat{a}_0^2$ which clearly $\to -\infty$ as $\hat{a}_0 \to 0$ or as $\hat{a}_0 \to \infty$. The maximum value of this expression is $-4(C/6)^{\frac{3}{4}} \approx -1.973$ which occurs when $\hat{a}_0 = (C/6)^{\frac{3}{8}} \approx 0.702$ and consequently we have the result that in the classical Taylor problem the the most unstable mode has wavenumber $\approx 0.702T^{\frac{3}{16}}$. As the scaled crossflow increases then $Re(\hat{\Omega}_1)$ is greatest when $\hat{a}_0 = O(\hat{\lambda}^{-\frac{1}{5}})$ and $|\hat{\Omega}_1| = O(\hat{\lambda}^{\frac{2}{5}})$. Furthermore, for a fixed $\hat{a}_0$, as $\hat{\lambda}$ increases so the overall vortex growth rate diminishes and crossflow is seen to have a stabilising effect. It is clear that a revised solution structure is needed when the second term in Gr becomes comparable with the first, i.e. when $\hat{\Omega}_1 = O(T^{\frac{1}{8}})$ or $\hat{\lambda} = O(T^{\frac{3}{16}})$.

We therefore define

$$\lambda = T^{\frac{1}{2}} \hat{\hat{\lambda}}, \qquad (25a)$$

and seek the wavenumber of the most unstable vortex for this case. We find that for a large range of wavenumbers, specifically $1 \ll a \ll T^{\frac{1}{4}}$, the leading order component of the normal velocity field, say $\hat{\hat{V}}_0$, is given by the solution of a Whittaker–type equation (see **[15]**). The disturbance structure is confined to lie within a thin zone adjacent to $y = 0$ whose precise

depth is a function of the wavenumber. However for all vortex wavenumbers $1 \ll a \ll T^{\frac{1}{4}}$ the leading order disturbance forms are determined by the solution of a common equation which may be solved analytically in terms of the modified Bessel function K_ν. In order to satisfy both the no-slip conditions at the wall and suitable decay conditions at the edge of the thin zone it is found that the leading order growth rate of the vortices is $\hat{\hat{\Omega}}_0 T^{\frac{1}{2}}$ where $\hat{\hat{\Omega}}_0$ is a solution of the equation

$$K_\mu\left(\hat{\hat{\Omega}}_0/i\hat{\hat{\lambda}}\right) = 0, \qquad \mu^2 = \tfrac{1}{4} + \hat{\hat{\lambda}}^{-2}. \tag{25b}$$

The dependence of $\hat{\hat{\Omega}}_0$ upon $\hat{\hat{\lambda}}$ is shown in Figure 2. When $\hat{\hat{\lambda}} = 0$ vortex modes with wavenumber $1 \ll a \ll T^{\frac{1}{4}}$ have growth rate $\hat{\hat{\Omega}}_0 = 1$. As $\hat{\hat{\lambda}}$ increases so the growth rate diminishes. Indeed, as $\hat{\hat{\lambda}} \to 1/\sqrt{2}$ so $\hat{\hat{\Omega}}_0 \to 0 - i/\sqrt{2}$ and all the modes with wavenumber within $1 \ll a \ll T^{\frac{1}{4}}$ are made neutral. Again therefore, just as in the Görtler problem, crossflow is seen to have a stabilising influence and at crossflows of greater than $Re^{-1}T^{\frac{1}{2}}/\sqrt{2}$ the flow configuration is stable to vortices of any wavenumber $\ll T^{\frac{1}{4}}$.

One question we have not addressed here is that concerning the structure of these vortex modes as the crossflow parameter $\hat{\hat{\lambda}}$ increases through the critical value $1/\sqrt{2}$. Indeed when $\hat{\hat{\lambda}} = 1/\sqrt{2}$ precisely then the formal solution for $\hat{\hat{V}}_0$ develops a singularity and the disturbance structure requires a critical layer type zone in order to smooth this out. This situation has similarities with the scenario discussed by Blackaby & Choudhari [**17**] who were concerned with inviscid vortex modes in slightly three–dimensional boundary layers and how such perturbations are related to the Taylor–Goldstein equation for stratified shear flows. These authors found solutions for which the leading order 'vertical' (i.e. normal to wall) velocity component of the inviscid disturbance becomes infinite at the critical layer and they asserted that theirs was the first case in which this type of behaviour has been found. We also believe this to be the case and suggest that we have here another problem where this phenomenon is seen.

The discussion above implies that for a large range of vortex wavenumbers all modes have the same leading order growth rate. However, such vortices are not the fastest growing ones as further investigation reveals that within the $O(T^{\frac{1}{4}})$ wavenumber regime we have the unique most unstable mode. In order to demonstrate this we define the crossflow parameter $\hat{\hat{\lambda}}$ as in (25*a*) and look for disturbances of wavenumber $a = \tilde{a}T^{\frac{1}{4}}$. If we let

$$E_4 \equiv \exp\left\{i\tilde{a}T^{\frac{1}{4}}z + T^{\frac{1}{2}}\int^t \left(\tilde{\Omega}(t) + \ldots\right)dt\right\}, \tag{26a}$$

and seek solutions of the type

$$(U, V, W, P) = (\tilde{U} + \ldots, T^{\frac{1}{2}}\tilde{V} + \ldots, T^{\frac{1}{2}}\tilde{W} + \ldots, T^{\frac{3}{4}}\tilde{P} + \ldots)E_4, \tag{26b}$$

where $\tilde{U}, \tilde{V}, \tilde{W}, \tilde{P}$ are functions of the scaled co-ordinate $\phi = T^{-\frac{1}{4}}y$ we find that $\tilde{U}$ and $\tilde{V}$ are given by the coupled viscous equations

$$\left(\frac{d^2}{d\phi^2} - 1 - \frac{\tilde{\Omega}}{\tilde{a}^2} - \frac{i\hat{\hat{\lambda}}\phi}{\tilde{a}^2}\right)\left(\frac{d^2}{d\phi^2} - 1\right)\tilde{V} = \tilde{a}^4\tilde{U}, \tag{27a}$$

$$\left(\frac{d^2}{d\phi^2} - 1 - \frac{\tilde{\Omega}}{\tilde{a}^2} - \frac{i\hat{\hat{\lambda}}\phi}{\tilde{a}^2}\right)\tilde{U} = -\tilde{V}, \tag{27b}$$

subject to the conditions that $\tilde{U} = \tilde{V} = d\tilde{V}/d\phi = 0$ at $\phi = 0$ and as $\phi \to \infty$. As would be expected in view of the scalings, these equations are essentially identical to those relevant to determining the right-hand branch of the neutral curve, (11).

We solved (27) for the (in general complex–valued) growth term $\tilde{\Omega}$ as a function of the crossflow parameter $\hat{\hat{\lambda}}$ and scaled wavenumber $\tilde{a}$ and the results of these calculations are summarised in figure 3. This figure shows that for $\hat{\hat{\lambda}} < 1/\sqrt{2}$ the vortex is unstable over a finite range of wavenumbers including those with $\tilde{a} \to 0$ and the extent of this unstable range decreases with increasing $\hat{\hat{\lambda}}$. Indeed when $\hat{\hat{\lambda}}$ reaches the value $1/\sqrt{2}$ the mode with $\tilde{a} = 0$ is made neutral in agreement with the results of the analysis relevant to modes with wavenumber $1 \ll a \ll T^{\frac{1}{4}}$. For increasing $\hat{\hat{\lambda}}$ beyond $1/\sqrt{2}$ the unstable band reduces further until when $\hat{\hat{\lambda}} \approx 0.942$ the whole wavenumber spectrum is made stable.

The results given in figure 3 also show the dependence of the most unstable vortex wavenumber upon the crossflow $\hat{\hat{\lambda}}$. If we denote the wavenumber of this most unstable mode by $\tilde{a}_u$ then as $\hat{\hat{\lambda}} \to 0$ so $\tilde{a}_u \to 0$ since we know from our previous workings that as the crossflow diminishes so the most unstable vortex reverts to within the $O(T^{\frac{3}{16}})$ regime. In addition, $\tilde{a}_u$ is not a monotonic function of $\hat{\hat{\lambda}}$ and as $\hat{\hat{\lambda}} \to\approx 0.942$ it is the mode with wavenumber $\tilde{a} \approx 0.103$ which is the last to be made stable.

The calculations in this and the preceding section give related but not identical information. The work of §3 may be regarded as describing the stability properties of a vortex of specified, large wavenumber. In particular, the solution of equations (11) accounts for the dependence of the neutral Taylor number on the size of the imposed crossflow whereas the work of this section describes how crossflow affects the stability of vortices within a flow at a specified large Taylor number. We notice how the most unstable vortex has wavenumber $O(T^{\frac{3}{16}})$ in the absence of crossflow; with increasing crossflow this fastest growing mode moves into the $O(T^{\frac{1}{4}})$ wavenumber regime. At this stage the right hand neutral branch of the stability curve is also within the $O(T^{\frac{1}{4}})$ range so that we can see that as the crossflow component increases from zero so the location of the most unstable linearised mode moves out into the asymptotic regime containing the right hand branch. This contrasts with the situation in the Görtler case where, for a large Görtler number G and in the absence of crossflow the right hand branch is at an $O(G^{\frac{1}{4}})$ wavenumber whilst

the fastest growing mode has wavenumber $O(G^{\frac{1}{5}})$, [8]. As the crossflow increases the most unstable vortex remains within the $O(G^{\frac{1}{5}})$ regime and the right hand neutral branch moves into this regime so that it is a mode of wavenumber $O(G^{\frac{1}{5}})$ which is the last to be stabilised.

5. Conclusions

In this work we have studied how the imposition of a small crossflow velocity component affects the linear stability characteristics of vortices in high Taylor number flow. In particular we have identified the most unstable vortex, illustrated that for large wavenumbers a crossflow of size $O(Re^{-1}a^2)$ is sufficient to completely stabilise the mode and proved that crossflow of $O(Re^{-1}T^{\frac{1}{2}})$ makes the flow immune to unstable vortices irrespective of their wavenumber. In some ways it is this last result which is the more significant for it shows that in many flows where an appreciable degree of three-dimensionality is present the Taylor vortex mechanism is likely to be unimportant compared with other instability forms.

Our results may be compared with those of the equivalent Görtler or Dean problem in the following way. Suppose that instead of having the inner cylinder of our geometry moving with speed U_0 it is at rest and an azimuthal pressure gradient is imposed of sufficient strength so as to induce an $O(U_0)$ fluid speed across $0 \leq y \leq 1$. The analysis of [8] suggests that in the absence of crossflow the most unstable vortex has wavenumber $O(T^{\frac{1}{5}})$. In addition a crossflow of size $O(Re^{-1}T^{\frac{3}{5}})$ would be needed to completely stabilise the flow– a value which is seen to be greater than that in the Taylor problem considered here. This suggests that in situations which are composed partly of a Taylor–type situation and partly a Dean–type then it the latter component which is dominant in determining the stability characteristics when crossflow is present. However, there will still be circumstances in which the structure presented in this paper is important and such cases include spin–up configurations of the type studied by Otto [2]. In that problem a cylinder at rest in an incompressible fluid is given an impulsive angular velocity. A Rayleigh layer is set up and this flow is prone to vortices which are identical in character to those examined here. If a basic flow component is added along the axis of the cylinder (so this could model a spinning cylindrical object moving through a fluid) this axial flow may be determined using the method of Glauert & Lighthill, [18].

This applicability of our analysis to other flow problems is an attraction of our working. The flow configuration chosen at the outset was deliberately taken to be very simple but it does capture most of the essential features of a wide class of flows over rotating bodies whenever the inviscid Rayleigh criterion for instability is most violated at the body surface. Not only can our analysis be extended to other geometries but it may also be easily adapted

to allow for vortex motions modulated in the azimuthal direction. Consequently it has the potential of proving to be useful in a number of related problems.

There are several directions in which our calculations might be extended. The most obvious of these is the question of nonlinearity. In order to give a definitive account of crossflow effects it would be necessary to allow for other than infinitesimal vortex modes by pursuing weakly nonlinear or strongly nonlinear computations. Equivalent work for the Görtler problem is reported in **[10, 11, 19]** where it is shown that weak nonlinearity stabilises near neutral but linearly growing modes to give non–zero finite amplitude vortex states. Inclusion of full nonlinearity tends to suggest that the vortices break down in a finite distance singularity which would be the analogue of a finite time singularity in the present work. In addition, of theoretical interest would be a rigorous analysis of the critical layer structure which arises just as vortices of wavenumber $1 \ll a \ll T^{\frac{1}{4}}$ near neutrality as the crossflow approaches $Re^{-1}T^{\frac{1}{2}}/\sqrt{2}$. This analysis would certainly be required in order to investigate nonlinear configurations near this cut–off. However the practical significance of such considerations is far from clear at this juncture for at this crossflow size we have shown that there is still a wide range of higher wavenumber vortices which are linearly unstable. Indeed it is not until that the axial flow becomes greater than $\approx 0.942Re^{-1}T^{\frac{1}{2}}$ that infinitesimal Taylor vortices of arbitrary wavenumber are made stable.

We are grateful to Dr. J.P. Denier for his valuable comments on an earlier draft of this work. The research of the first author was supported by the National Aeronautics and Space Administration under NASA contract No. NAS1–19480 while he was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23681, USA.

References

1. P. Hall, *J. Fluid Mech.* **124** (1982) 475–494.

2. S.R. Otto, *IMA J. Appl. Maths.*, in press (1993).

3. P. Hall, *J. Inst. Maths. Applics.* **29** (1982) 173–196.

4. P. Hall and W.D. Lakin, *Proc. R. Soc. Lond. A* **415** (1988) 421–444.

5. A.P. Bassom and P. Hall, *Stud. appl. Math.* **81** (1989) 185–219.

6. J.P. Denier, *IMA J. Appl. Maths.* **49** (1992) 15–33.

7. P. Hall, *Proc. R. Soc. Lond. A* **399** (1985) 135–152.

8. J.P. Denier, P. Hall and S.O. Seddougui, *Phil. Trans. R. Soc. Lond. A* **335** (1991) 51–85.

9. A.P. Bassom and P. Hall, *J. Fluid Mech.* **232** (1991) 647–680.

10. A.P. Bassom and S.R. Otto, *J. Fluid Mech.* **249** (1993) 597–618.

11. S.R. Otto and A.P. Bassom, *Stud. appl. Math.*, in press (1994).

12. R. C. DiPrima and A. Pridor, *Proc. R. Soc. Lond. A* **366** (1979) 555–573.

13. S. Goldstein, *Proc. Camb. Phil. Soc.* **33** (1937) 41–61.

14. R. J. Cornish, *Proc. R. Soc. Lond. A* **140** (1933) 227–240.

15. M. Abramowitz and I.A. Stegun, *Handbook of Mathematical Functions*, (Dover, New York 1972).

16. P.G. Drazin and W.H. Reid, *Hydrodynamic Stability*, (CUP, Cambridge 1981).

17. N.D. Blackaby and M. Choudhari, *Proc. R. Soc. Lond. A* **440** (1993) 701–710.

18. M.B. Glauert and M.J. Lighthill, *Proc. R. Soc. Lond. A* **230** (1955) 188–203.

19. J.P. Denier and P. Hall, *J. Fluid Mech.* **247** (1993) 1–16.

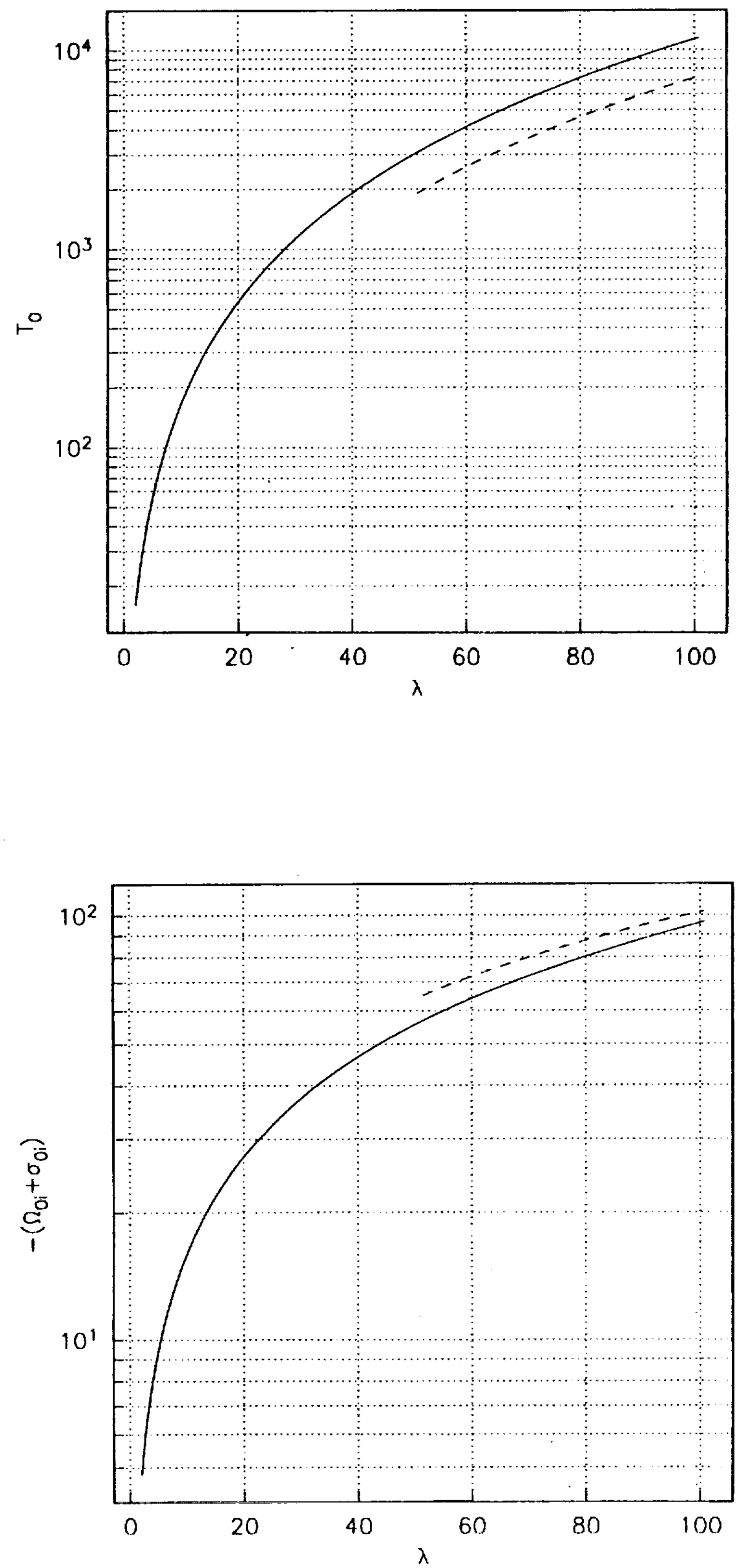


Figure 1. Solution of equations (11) in order to determine the location of the right–hand neutral branch as a function of crossflow parameter $\tilde{\tilde{\lambda}}$; *a)* T_0; *b)* $\mathrm{Im}(\tilde{\Omega}_0 + \tilde{\sigma}_0)$. The dotted lines on the figures denote the asymptotic forms (12) and (16).

Figure 2. The dependences of $Re(\hat{\hat{\Omega}}_0)$ (solid line) and $Im(\hat{\hat{\Omega}}_0)$ (broken line) upon the imposed crossflow $\hat{\hat{\lambda}}$. Also shown is the asymptotic form $\hat{\hat{\Omega}}_0 = 1 - 0.928\hat{\hat{\lambda}}^{\frac{2}{3}}(1 + i\sqrt{3}) + \ldots$ which is valid for small scaled crossflow values.

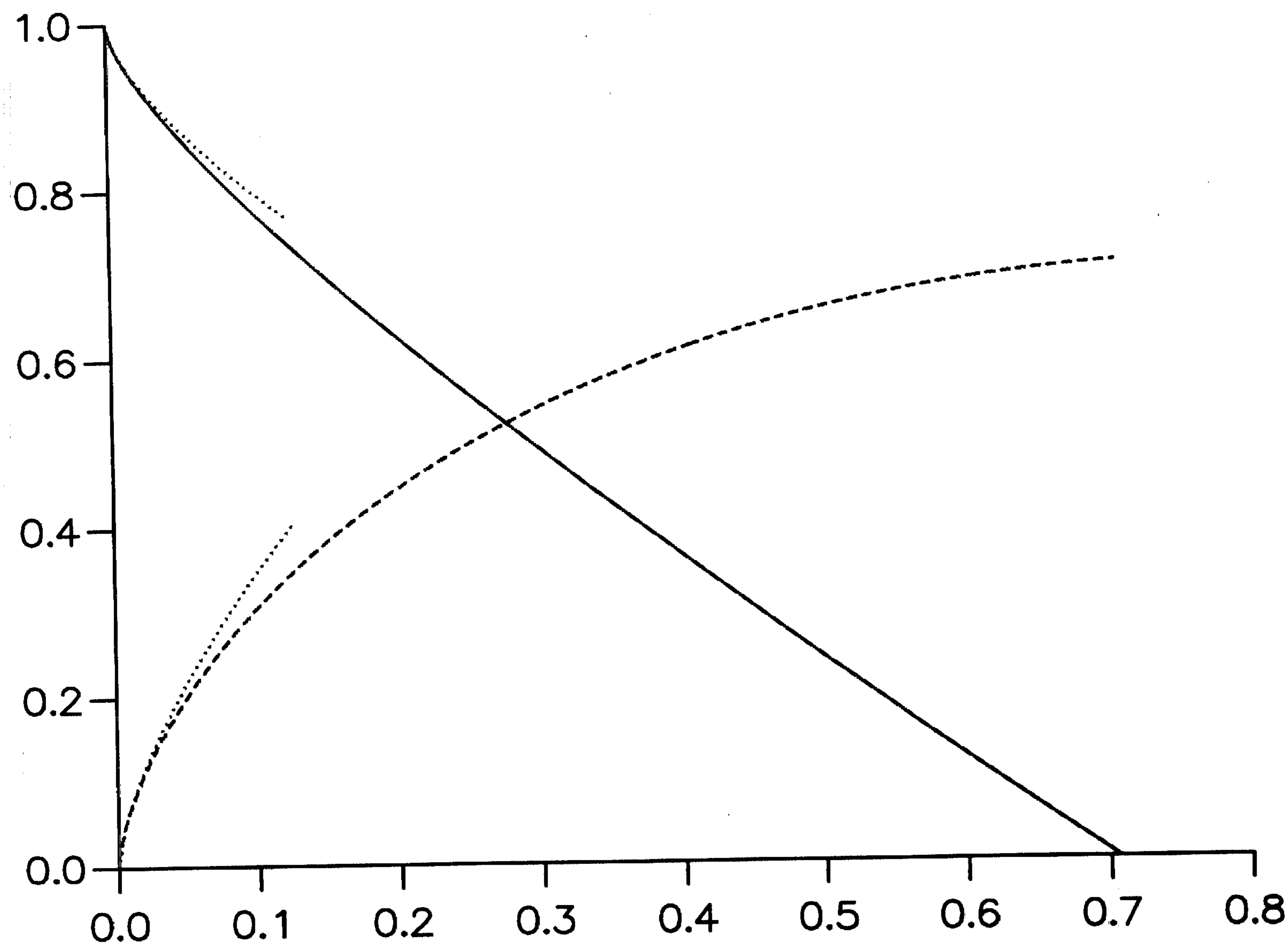
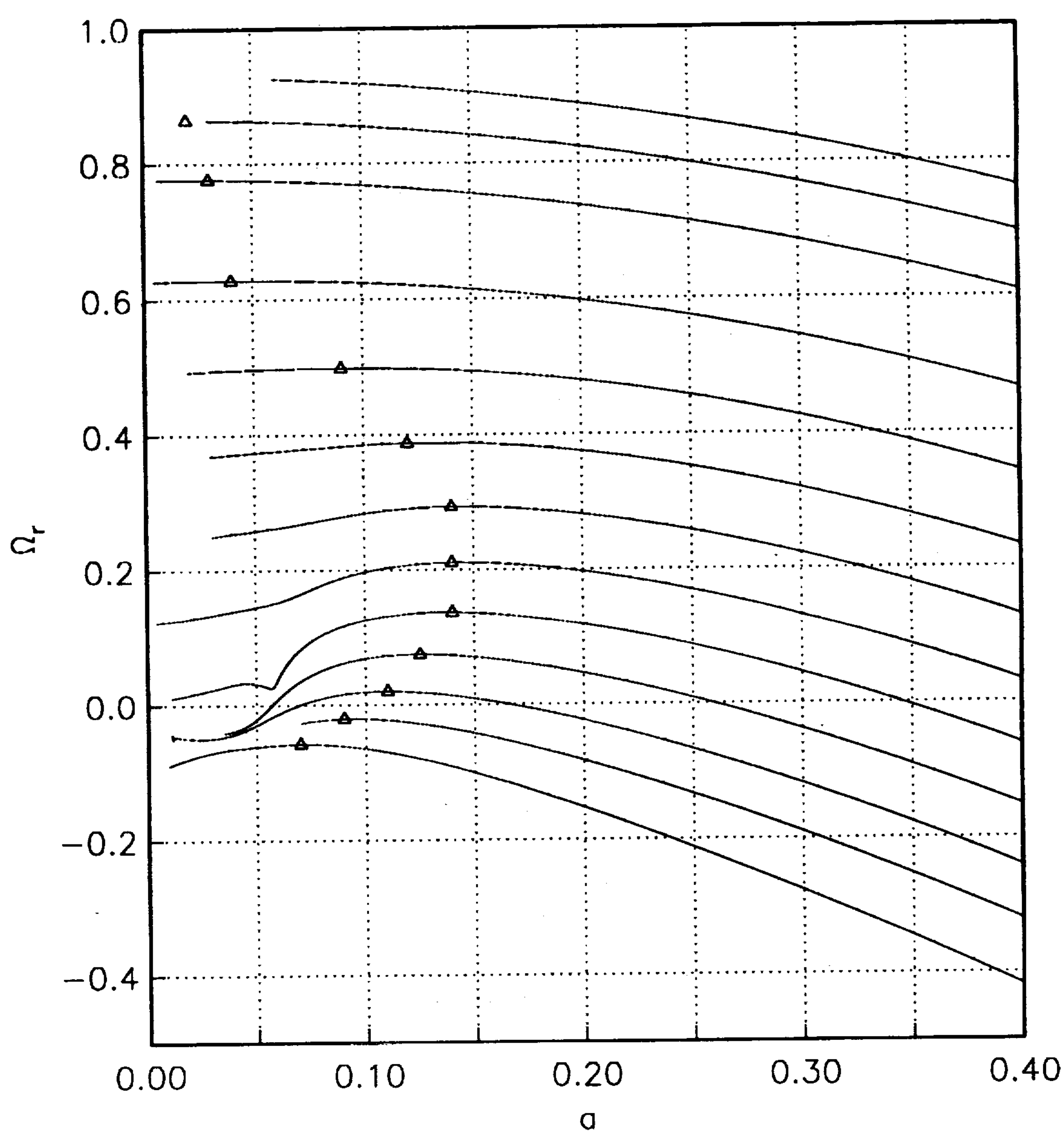


Figure 3. Solution of viscous equations (27) to determine leading order vortex growth rate $Re(\tilde{\Omega})T^{\frac{1}{2}}$ as a function of vortex wavenumber $\tilde{a}T^{\frac{1}{4}}$ and crossflow $\hat{\hat{\lambda}}T^{\frac{1}{2}}$. The graphs correspond to crossflow values $\hat{\hat{\lambda}} = 0.02$, 0.05, 0.1 and thereafter at intervals of 0.1 up to and including $\hat{\hat{\lambda}} = 1.1$. The Δ symbols indicate the locus of the most unstable vortex wavenumber.



REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
June 1993

3. REPORT TYPE AND DATES COVERED
Contractor Report

4. TITLE AND SUBTITLE
THE EFFECT OF CROSSFLOW ON TAYLOR VORTICES: A MODEL PROBLEM

5. FUNDING NUMBERS
C NAS1-19480
WU 505-90-52-01

6. AUTHOR(S)
S. R. Otto
Andrew P. Bassom

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Institute for Computer Applications in Science and Engineering
Mail Stop 132C, NASA Langley Research Center
Hampton, VA 23681-0001

8. PERFORMING ORGANIZATION REPORT NUMBER
ICASE Report No. 93-33

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23681-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA CR-191487
ICASE Report No. 93-33

11. SUPPLEMENTARY NOTES
Langley Technical Monitor: Michael F. Card
Final Report
Submitted to Quarterly Journal of Mechanics and Applied Mathematics

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited

Subject Category 34

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A number of practically relevant problems involving the impulsive motion or the rapid rotation of bodies immersed in fluid are susceptible to vortex-like instability modes. Depending upon the configuration of any particular problem the stablity properties of any high-wavenumber vortices can take on one of two distinct forms. One of these is akin to the structure of Görtler vortices in boundary layer flows whilst the other is similar to the situation for classical Taylor vortices.

Both the Görtler and Taylor problems have been extensively studied when crossflow effects are excluded from the underlying base flows. Recently, studies have been made concerning the influence of crossflow on Görtler modes and here we use a linearised stability analysis to examine crossflow properties for the Taylor mode. This work allows us to identify the most unstable vortex as the crossflow component increases and we show how, like the Görtler case, only a very small crossflow component is required in order to completely stablise the flow. Our investigation forms the basis for an extension to the nonlinear problem and is of potential applicability to a range of pertinent flows.

14. SUBJECT TERMS
Taylor vortices, crossflow

15. NUMBER OF PAGES
21

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT
Unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT

20. LIMITATION OF ABSTRACT

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std Z39-18
298-102

☆U.S. GOVERNMENT PRINTING OFFICE: 1993 - 728-064/86032